Northview High School
AB Calculus AP Prerequisites Packet

To: AP Calculus AB Students and Parents
From: Tori Hedden, Blake Pinto

The AP Course: AP Calculus AB is a college level course covering material traditionally taught in the first semester of college calculus.
The Prerequisite Packet: Students need a strong foundation to be ready for the rigorous work required throughout the term. Completing the prerequisite packet should prepare you for the material to be taught in the course. This packet consists of material studied during Algebra II and Precalculus. Students should anticipate working approximately 3 hours to complete it properly. This packet includes

- A "Toolkit of Functions"; you should be familiar with each of the graphs.
- A formula and identities section.
- A unit circle template with which to practice your unit circle.
- A list of skills that you will need for AP Calculus.
- Calculus Prerequisite Problems - please show all work.

In preparation for the AP test in May, students need to begin showing all work with logical steps. Do not list only an answer. Work neatly and in an organized fashion. Please expect a test within the first few days of the school year over this material. You may ask questions over the packet and any other pre-requisite information when we return in August. We will begin new material by the 4th day of school, so it is imperative that these skills are mastered in order to be successful moving forward in the course.

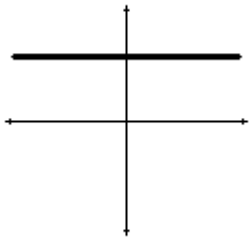
Calculators: Students enrolled in AP Calculus AB will be using a graphing calculator throughout the course. A graphing calculator is required on the AP test. A list of acceptable calculators for the AP test is available online at www.apcentral.collegeboard.com. Considering that half of the AP exam is without calculators, expect about half of your quizzes and tests throughout the course to be without calculators.

Toolkit of Functions

Students should know the basic shape of these functions and be able to graph their transformations without the assistance of a calculator.

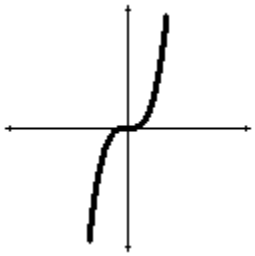
Constant

$f(x) = a$



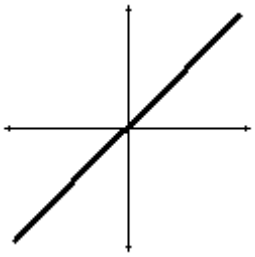
Cubic

$f(x) = x^3$



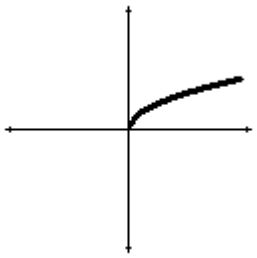
Identity

$f(x) = x$



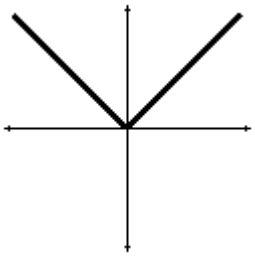
Square Root

$f(x) = \sqrt{x}$



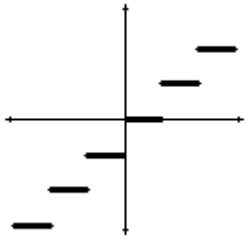
Absolute Value

$f(x) = | x |$



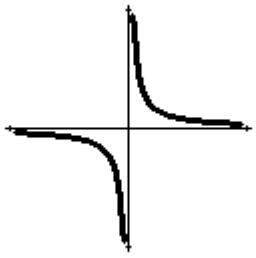
Greatest Integer

$f(x) = [x]$



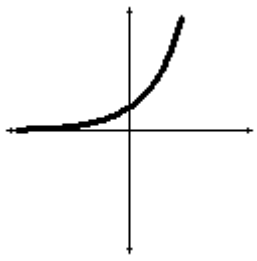
Reciprocal

$f(x) = \frac{1}{x}$



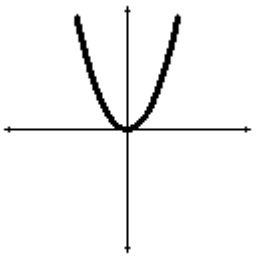
Exponential

$f(x) = a^x$



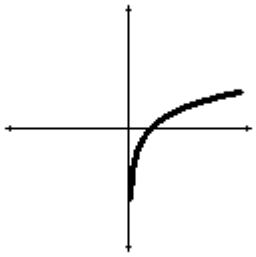
Quadratic

$f(x) = x^2$



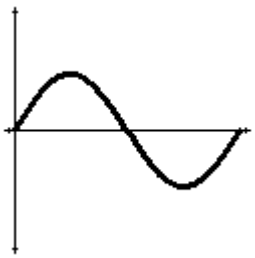
Logarithmic

$f(x) = \ln x$

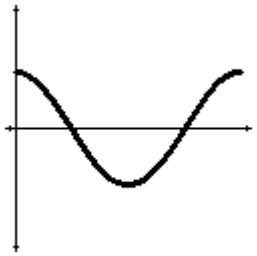


Trig Functions

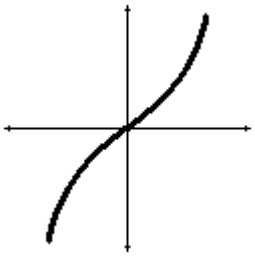
$f(x) = \sin x$



$f(x) = \cos x$



$f(x) = \tan x$



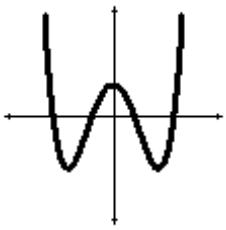
Polynomial Functions:

A function P is called a polynomial if $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$
Where *n* is a nonnegative integer and the numbers a_0, a_1, a_2, … a_n are constants.

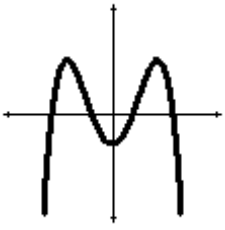
Even degree

Leading coefficient sign

Positive



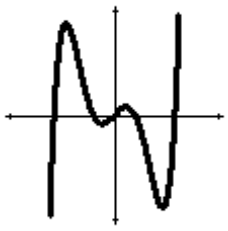
Negative



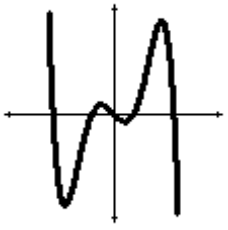
Odd degree

Leading coefficient sign

Positive



Negative



- Number of roots equals the degree of the polynomial.
- Number of x intercepts is less than or equal to the degree.
- Number of "bends" is less than or equal to (degree – 1).

Formulas and Identities

Trig Identities:

Reciprocal Identities:

$$\csc A = \frac{1}{\sin A} \qquad \sec A = \frac{1}{\cos A} \qquad \cot A = \frac{1}{\tan A}$$

Quotient Identities: $\tan A = \frac{\sin A}{\cos A}$ $\cot A = \frac{\cos A}{\sin A}$

Pythagorean Identities:

$\sin^2 A + \cos^2 A = 1$ $\tan^2 A + 1 = \sec^2 A$ $1 + \cot^2 A = \csc^2 A$

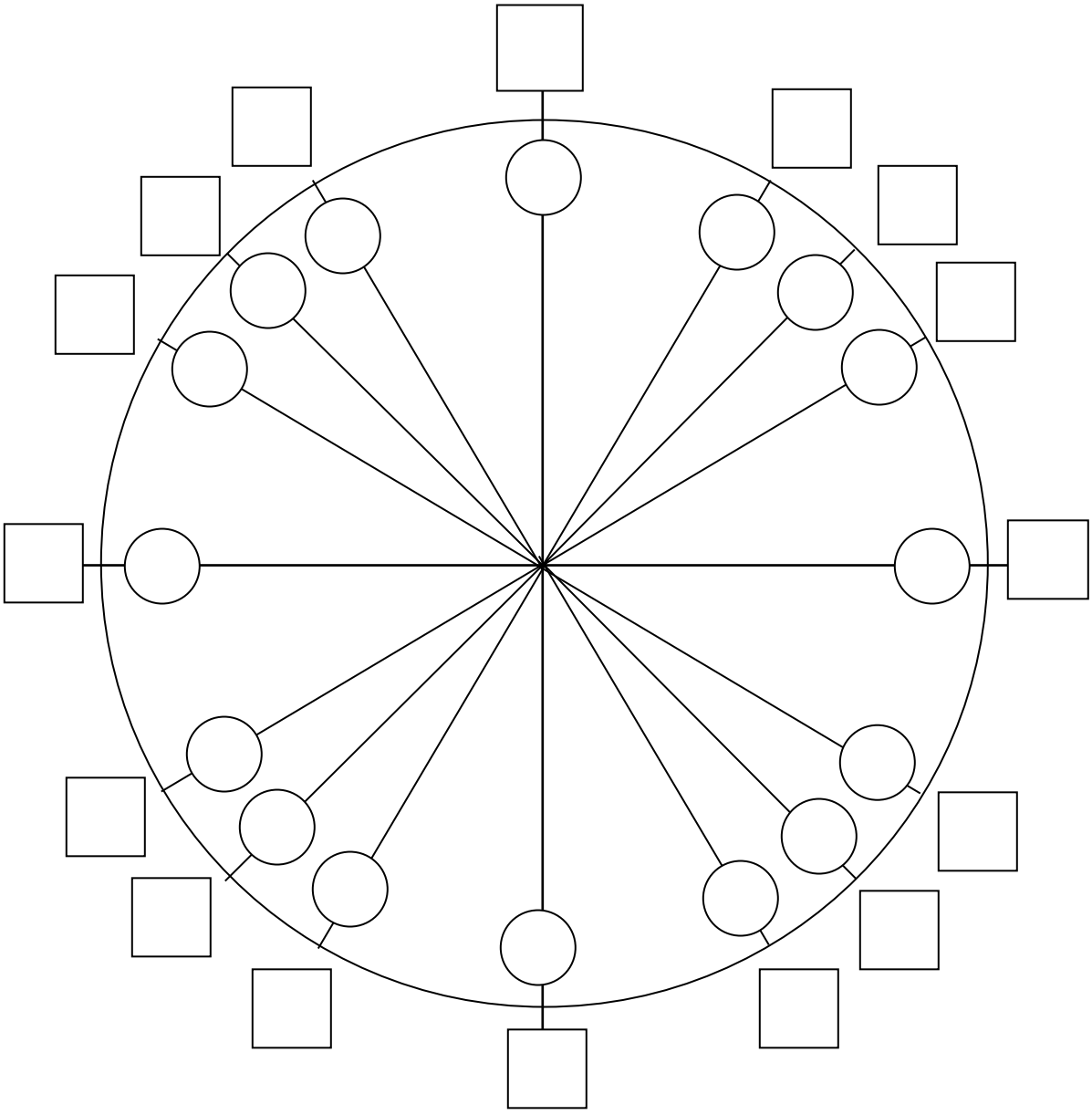
Geometric Formulas:

Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$ Area of a triangle: $A = \frac{1}{2}bh$

Area of an equilateral triangle: $A = \frac{\sqrt{3}}{4}s^2$

Area if a circle: $A = \pi r^2$ Circumference of a circle: $C = 2\pi r$ or $C = d\pi$

Unit Circle – Degrees and Radians



Place degree measures in the circles.

Place radian measure in the squares.

Place $(\cos\theta, \sin\theta)$ in parenthesis outside the square.

Place $\tan\theta$ outside the parenthesis.

$\tan\theta$ = _____

$\cot\theta$ = _____

$\csc\theta$ = _____

$\sec\theta$ = _____

SKILLS NEEDED FOR CALCULUS

I. Algebra:

*A. Exponents (operations with integer, fractional, and negative exponents)
*B. Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)
C. Rationalizing (numerator and denominator)
*D. Simplifying rational expressions
*E. Solving algebraic equations and inequalities (linear, quadratic, higher order using synthetic division, rational, radical, and absolute value equations)
F. Solving simultaneous equations

II. Graphing and Functions

*A. Lines (intercepts, slopes, write equations using point-slope and slope intercept, parallel, perpendicular, distance and midpoint formulas)
B. Conic Sections (circle, parabola, ellipse, and hyperbola)
*C. Functions (definition, notation, domain, range, inverse, composition)
*D. Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric. piece-wise, inverse functions)
E. Tests for symmetry: odd, even

III. Geometry

A. Pythagorean Theorem
B. Area Formulas (Circle, polygons, surface area of solids)
C. Volume formulas
D. Similar Triangles

****IV. Logarithmic and Exponential Functions***

*A. Simplify Expressions (Use laws of logarithms and exponents)
*B. Solve exponential and logarithmic equations (include ln as well as log)
*C. Sketch graphs
*D. Inverses

****V. Trigonometry***

**A. Unit Circle (definition of functions, angles in radians and degrees)
B. Use of Pythagorean Identities and formulas to simplify expressions and prove identities
*C. Solve equations
*D. Inverse Trigonometric functions
E. Right triangle trigonometry
*F. Graphs

VI. Limits

A. Concept of a limit
B. Find limits as x approaches a number and as x approaches ∞

* A solid working foundation in these areas is very important.

Calculus Prerequisite Problems

Work the following problems on your own paper. Show all necessary work.

I. Algebra

A. Exponents: 1) $\dfrac{\left(8x^3yz\right)^{1/3}(2x)^3}{4x^{1/3}\left(yz^{2/3}\right)^{-1}}$

B. Factor Completely:

2) $9x^2 + 3x - 3xy - y$ (use grouping) 3) $64x^6 - 1$ *Hint: Factor as difference of squares first, then as the sum and difference of cubes second.*

4) $42x^4 + 35x^2 - 28$ 5) $15x^{5/2} - 2x^{3/2} - 24x^{1/2}$ Hint: Factor GCF $x^{1/2}$ first.

6) $x^{-1} - 3x^{-2} + 2x^{-3}$ Hint: Factor out GCF x^{-3} first.

C. Rationalize denominator / numerator:

7) $\dfrac{3-x}{1-\sqrt{x-2}}$ 8) $\dfrac{\sqrt{x+1}+1}{x}$

D. Simplify the rational expression:

9) $\dfrac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

E. Solve algebraic equations and inequalities

10. – 11. *Use synthetic division to help factor the following, state all factors and roots.*

10) $p(x) = x^3 + 4x^2 + x - 6$ 11) $p(x) = 6x^3 - 17x^2 - 16x + 7$ (remember the rational root theorem! $\frac{p}{q}$)

12) <u>*Explain*</u> why $\frac{3}{2}$ cannot be a root of $f(x) = 4x^5 + cx^3 - dx + 5$, where c and d are integers. (hint: You can look at the possible rational roots.)

13) <u>*Explain*</u> why $f(x) = x^4 + 7x^2 + x - 5$ must have a root in the interval [0, 1], ($0 \le x \le 1$) Check the graph and use signs of f(0) and f(1) to justify your answer.

F. Solve the system. Solve the system algebraically and then check the solution by graphing each function and using your calculator to find the points of intersection.

14) $x - y + 1 = 0$
$y - x^2 = -5$

15) $x^2 - 4x + 3 = y$
$-x^2 + 6x - 9 = y$

II. Graphing and Functions:

A. Linear graphs: *Write the equation of the line described below.*

16) Passes through the point (2, -1) and has slope $-\frac{1}{3}$.

17) Passes through the point (4, - 3) and is perpendicular to $3x + 2y = 4$.

18) Passes through (-1, - 2) and is parallel to $y = \frac{3}{5}x - 1$.

B. Functions: *Find the domain and range of the following.*

Note: domain restrictions - denominator ≠ 0, argument of a log or ln > 0,
radicand of even index must be ≥ 0
range restrictions- reasoning, if all else fails, use graphing calculator

19) $y = \frac{3}{x - 2}$ 20) $y = \log(x - 3)$ 21) $y = x^4 + x^2 + 2$

22) $y = \sqrt{2x - 3}$ 23) $y = |x - 5|$ 24) domain only: $y = \frac{\sqrt{x+1}}{x^2 - 1}$

25) Given f(x) below, graph over the domain [-3, 3], what is the range?

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 1 & \text{if } -1 \leq x < 0 \\ x - 2 & \text{if } x < -1 \end{cases}$$

Find the composition /inverses as indicated below.

Let **$f(x) = x^2 + 3x - 2$** **$g(x) = 4x - 3$** **$h(x) = \ln x$** **$w(x) = \sqrt{x - 4}$**

26) $g^{-1}(x)$ 27) $h^{-1}(x)$ 28) $w^{-1}(x)$, for $x \geq 4$ 29) f(g(x)) 30) h(g(f(1)))

31) Does $y = 3x^2 - 9$ have an inverse function? Explain your answer.

Let **$f(x) = 2x$, $g(x) = -x$, and $h(x) = 4$,** find

32) $(f \circ g)(x)$ 33) $(f \circ g \circ h)(x)$

34) Let $s(x) = \sqrt{4 - x}$ and $t(x) = x^2$, find the domain and range of $(s \circ t)(x)$.

C. Basic Shapes of Curves:

Sketch the graphs. You may use your graphing calculator to verify your graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

35) $y = \sqrt{x}$ 36) $y = \ln x$ 37) $y = \frac{1}{x}$ 38) $y = |x - 2|$

39) $y = \frac{1}{x - 2}$ 40) $y = \frac{x}{x^2 - 4}$ 41) $y = 2^{-x}$

42) $f(x) = \begin{cases} \sqrt{25 - x^2} & \text{if } x < 0 \\ \frac{x^2 - 25}{x - 5} & \text{if } x \geq 0,\ x \neq 5 \\ 0 & \text{if } x = 5 \end{cases}$

D. Even, Odd, Tests for Symmetry:

Id*entify as odd, even , or neither and justify you're answer.* *To justify your answer you must show substitution using -x ! It is not enough to simply check a number*.

Even: $f(x) = f(-x)$ Odd: $f(-x) = -f(x)$

43) $f(x) = x^3 + 3x$ 44) $f(x) = x^4 - 6x^2 + 3$ 45) $f(x) = \frac{x^3 - x}{x^2}$

46) $f(x) = \sin 2x$ 47) $f(x) = x^2 + x$ 48) $f(x) = x(x^2 - 1)$

49) $f(x) = \frac{1 + |x|}{x^2}$

50) What type of function (even or odd) results from the product of two

even functions? odd functions?

Test for symmetry. Show substitution with variables to justify you're answer.

Symmetric to y axis: replace x with **-** x and relation remains the same.
Symmetric to x axis: replace y with **-** y and relation remains the same.
Origin symmetry: replace x with **-** x, y with **-** y and the relation is equivalent.

51) $y = x^4 + x^2$ 52) $y = \sin(x)$ 53) $y = \cos(x)$

54) $x = y^2 + 1$ 55) $y = \frac{|x|}{x^2 + 1}$

IV LOGARITHMIC AND EXPONENTIAL FUNCTIONS

A. Simplify Expressions:

56) $\log_4\left(\frac{1}{16}\right)$ 57) $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3\left(\frac{1}{27}\right)$ 58) $\log_9 27$

59) $\log_{125}\left(\frac{1}{5}\right)$ 60) $\log_w w^{45}$ 61) $\ln e$ 62) $\ln 1$ 63) $\ln e^2$

B. Solve equations:

64) $\log_6(x+3) + \log_6(x+4) = 1$ 65) $\log x^2 - \log 100 = \log 1$ 66) $3^{x+1} = 15$

V TRIGONOMETRY

A. Solve the Equations

67) $\cos^2 x = \cos x + 2, \quad 0 \le x \le 2\pi$

68) $\cos^2 x + \sin x + 1 = 0, \quad 0 \le x \le 2\pi$ (Hint: use your trig identities!)

B. Be able to do the following on your graphing calculator:

Be familiar with the CALC commands; value, root, minimum, maximum, intersect. You may need to zoom in on areas of your graph to find the information.
Answers should be accurate to 3 decimal places. Sketch graph.

69. – 72. Given the following function $f(x) = 2x^4 - 11x^3 - x^2 + 30x$.

69. Find all roots.

Note: Window x min: -10 x max: 10 scale 1
y min: - 100 y max: 60 scale 0

70. Find all local maxima.

71. Find all local minima.

A local maximum or local minimum is a point on the graph where there is a highest or lowest point within an interval such as the vertex of a parabola.

72. Find the following values: f (-1), f (2), f (0), f (.125)

73. Graph the following two functions and find their points of intersection using the intersect command on your calculator.

$y = x^3 + 5x^2 - 7x + 2$ and $y = .2x^2 + 10$ Window: x min : -10 x max: 10 scale 1
y min: - 10 y max: 50 scale 0

Answers: (Remember – you must show all of your work!)

1. $4x^{11/3}y^{4/3}z$ 2. $(3x + 1)(3x - y)$ 3. $(2x - 1)(4x^2 + 2x + 1)(2x + 1)(4x^2 - 2x + 1)$

4. $7(3x^2 + 4)(2x^2 - 1)$ 5. $x^{1/2}(3x - 4)(5x + 6)$

6. $x^{-3}(x - 2)(x - 1)$ 7. $1 + \sqrt{x - 2}$ 8. $\dfrac{1}{\sqrt{x+1}-1}$ 9. $\dfrac{x^2 - x + 1}{(x + 1)^2}$

10. $(x - 1)(x + 2)(x + 3)$; 1, -2, -3 11. $(x + 1)(2x - 7)(3x - 1)$; $-1, \dfrac{7}{2}, \dfrac{1}{3}$

12. not a possible rational root 13. f(0) = neg and f(1) = positive 14. (3, 4), (-2, -1)

15. (2, -1) , (3, 0) 16. $y = -\dfrac{1}{3}x - \dfrac{1}{3}$ 17. $y = \dfrac{2}{3}x - \dfrac{17}{3}$ 18. $y = \dfrac{3}{5}x - \dfrac{7}{5}$

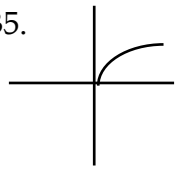
19. D: $x \neq 2$ R: $y \neq 0$ 20. $D: x > 3$ R:all reals 21. D: all reals R: $x \geq 2$

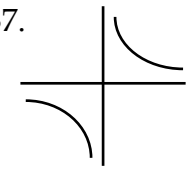
22. D: $x \geq 3/2$ R: $y \geq 0$ 23. D: all reals R: $y \geq 0$

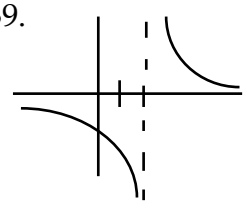
24. $x > -1$ and $x \neq 1$ 25. R: $-5 \leq y < -3$ or $0 \leq y \leq 3$ 26. $f^{-1}(x) = \dfrac{x+3}{4}$ 27. $h^{-1}(x) = e^x$

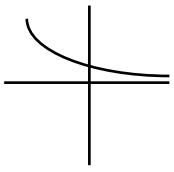
28. $y = x^2 + 4$ $x \geq 0$, 29. $f(g(x)) = 16x^2 - 12x - 2$ 30. ln 5 31. no, explain 32. $-2x$

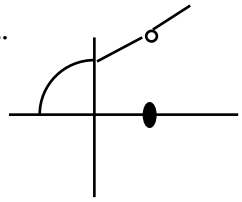
33. - 8 34. D: $-2 \leq x \leq 2$ R: $0 \leq y \leq 2$

35. 

37. 

39. 

41. 

42. 

You must show work on these! 43. odd 44. even 45. odd 46. odd 47. neither 48. odd

49. even 50. even, even 51. y axis 52. origin 53. y axis 54. x axis 55. y axis

56. - 2 57. –1 58. 3/2 59. – 1/3 60. 45 61. 1 62. 0 63. 2 64. - 1

65. x = 10, -10 66. $\dfrac{\log 15}{\log 3}$ - 1 67. π 68. $\dfrac{3\pi}{2}$ 69. - 1.5, 0 , 2 , 5

70. rel max. (1.07, 20.1) 71. rel. min (- .89, - 18. 48), (3.94, - 88)

72. f(-1) = - 18 73. 3 points of intersection (-5.77, 16.66) , (-.787, 10.124), (1.760, 10.620)